Iterative methods for nonlinear tomographic reconstruction

Computed tomography (CT) scans, comprising X-ray measurements taken at several angles, are used broadly in fields such as medical imaging, archaeology and nondestructive material evaluation. Many commercial CT scanners produce images via a non-iterative procedure derived from a linear approximation of the measurement acquisition process, trading off reconstruction accuracy for computational efficiency. Iterative recovery methods, used in conjunction with improved measurement models, can provide high-quality reconstructions; however, they may require a prohibitive amount of iterations or X-ray measurements (samples), or even fail to converge entirely.



Figure 1: Empirical probability of signal recovery over 25 independent trials; lighter tiles indicate higher recovery probability. Comparison with baseline method from [2]. Image adapted from [1].

Our work [1] studies the computational and sample complexity of signal recovery from CT measurements, casting CT image recovery as the solution of a high-dimensional nonlinear optimization problem. Under appropriate statistical assumptions, we prove that a "low-complexity" iterative optimization method – namely, the subgradient method equipped with the Polyak step size – can lead to faster reconstruction from substantially fewer samples. In particular, the computational and sample complexities of our method scale polynomially with the intensity of the unknown image, while existing work [2] exhibits exponential scaling (see **Figure 1** above). We complement our study with several numerical experiments demonstrating the advantages of our method in a variety of imaging setups, including photon-counting CT with Poisson measurement noise.

References

- 1. Vasileios Charisopoulos and Rebecca Willett. 2024. Nonlinear tomographic reconstruction via nonsmooth optimization. arXiv:2407.12984.
- 2. Sara Fridovich-Keil, Fabrizio Valdivia, Gordon Wetzstein, Benjamin Recht, and Mahdi Soltanolkotabi. 2023. Gradient descent provably solves nonlinear tomographic reconstruction. arXiv:2310.03956.